Double beta decay in heavy deformed nuclei: what have we learned?

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Abstract:

The pseudo SU(3) approach is used to describe low lying states and BE(2) intensities of rare earth and actinide nuclei which are $\beta\beta$ decay candidates. The $\beta\beta$ half lives of some of these nuclei to the ground and excited states of the final ones are evaluated for both the two and zero neutrino emitting modes. The existence of selection rules which strongly restricts the decays is discussed. These restrictions represent a possible test of the model. Up to now the predictions are in good agreement with the available experimental data.

Resumen:

El formalismo pseudo SU(3) es usado para describir los estados de menor energía y sus intensidades BE(2) de núcleos en la región de las tierras raras y los actinídos que son candidatos a decaimientos $\beta\beta$. Las vidas medias de esta desintegración para algunos de estos núcleos al estado fundamental y estados excitados del núcleo hijo son evaluadas en los modos con y sin emisión de neutrinos. Se discute la existencia de reglas de selección que restringen fuertemente los decaimientos y representan una posible prueba para el modelo. Hasta el momento el acuerdo entre las predicciones y los datos experimentales disponibles es bueno.

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1 Introduction

The detection of the neutrinoless double beta decay ($\beta\beta_{0\nu}$) would imply an indisputable evidence of physics beyond the standard model and would be useful in order to select Grand Unification Theories[1]. Theoretical nuclear matrix elements are needed to convert experimental half-life limits, which are available for many $\beta\beta$-unstable isotopes[2], into constrains for particle physics parameters such as the effective Majorana mass of the neutrino and the contribution of right-handed currents to the weak interaction. The two neutrino mode of the double beta decay ($\beta\beta_{2\nu}$) is allowed as a second order process in the standard model. It has been detected in nine nuclei[2] and has served as a test of a variety of nuclear models. It is the best available proof we can impose to a nuclear model used to predict the $\beta\beta_{0\nu}$ matrix elements.

Many experimental groups have reported measurements of $\beta\beta_{2\nu}$ processes[2]. Nearly for all the cases the ground state (g.s.) to ground state ($0^+ \rightarrow 0^+$) decay was investigated. In direct-counting experiments the analysis of the sum-energy spectrum of the emitted electrons allowed the identification of the different $\beta\beta$-decay modes. The experimental research on the double beta decay of heavy deformed nuclei enjoys an increasing activity. Thus, the $\beta\beta_{2\nu}$ half-life of ^{238}U[3] and ^{150}Nd[2, 4, 5] have been measured and search is in progress for ^{244}Pu[6].

In previous papers [7, 8] we used the pseudo SU(3) shell model to evaluate the two neutrino double beta half lifes of eleven heavy deformed potential double beta emitters. We found good agreement with the available experimental information. The radiochemically measured ^{238}U decay [3] raised some expectations, given the experimental arrangement cannot discriminate between the different double beta decay modes, and some theoretical estimates[9] predicted similar decay ratios for both the 0ν and the 2ν modes. Our calculations for the two neutrino mode were consistent with the experimental result.

We have also calculated the zero neutrino matrix elements for six heavy deformed double beta emitters[10]. Using the upper limit for the neutrino mass we also estimated their $\beta\beta_{0\nu}$ half-lives. In the case of ^{238}U we found the zero neutrino half-life at least three orders of magnitude greater than the two neutrino one, giving strong support of the identification of the observed half-life as being the two neutrino double beta decay. The $\beta\beta_{2\nu}$ of ^{150}Nd to the ground and

excited states of ^{150}Sm has also been studied[11]. In the present paper we review the above mentioned calculations as well as the pseudo SU(3) formalism used in those works.

2 The pseudo SU(3) formalism

In the pseudo $SU(3)$ shell model coupling scheme[12] normal parity orbitals (η, l, j) are identified with orbitals of a harmonic oscillator of one quanta less $\tilde{\eta} = \eta - 1$. This set of orbitals with $\tilde{j} = j = \tilde{l} + \tilde{s}$, pseudo spin $\tilde{s} = 1/2$ and pseudo orbital angular momentum $\tilde{l}$ define the so called pseudo space. The orbitals with $j = \tilde{l} \pm 1/2$ are nearly degenerate. For configurations of identical particles occupying a single j orbital of abnormal parity a convenient characterization of states is made by means of the seniority coupling scheme.

The many particle states of n_α nucleons in a given shell η_α, $\alpha = \nu$ or π, can be defined by the totally antisymmetric irreducible representations $\{1^{n^N_\alpha}\}$ and $\{1^{n^A_\alpha}\}$ of unitary groups. The dimensions of the normal (N) parity space is $\Omega^N_\alpha = (\tilde{\eta}_\alpha + 1)(\tilde{\eta}_\alpha + 2)$ and that of the unique (A) space is $\Omega^A_\alpha = 2\eta_\alpha + 4$ with the constraint $n_\alpha = n^A_\alpha + n^N_\alpha$. Proton and neutron states are coupled to angular momentum J^N and J^A in both the normal and unique parity sectors, respectively. The wave function of the many-particle state with angular momentum J and projection M is expressed as a direct product of the normal and unique parity ones, as:

$$|JM> = \sum_{J^N J^A} [|J^N > \otimes |J^A >]^J_M \tag{1}$$

We are interested in the low energy states which have $J = 0, 2, 3, 4, 6$. In the normal subspace only pseudo spin zero configurations are taken into account. Additionally in the abnormal parity space only seniority zero configurations are taken into account. This simplification implies that $J^A_\pi = J^A_\nu = 0$. This is a very strong assumption quite useful in order to simplify the calculations. Its effects upon the present calculation are discussed below.

The double beta decay, when described in the pseudo SU(3) scheme, is strongly dependent on the occupation numbers for protons and neutrons in the normal and abnormal parity

states $n_\pi^N, n_\nu^N, n_\pi^A, n_\nu^A$[8]. These numbers are determined filling the Nilsson levels from below, as discussed in [8]. In particular the $\beta\beta_{2\nu}$ decay is allowed only if they fulfil the following relationships

$$\begin{array}{ll} n^A_{\pi,f} = n^A_{\pi,i} + 2 \ \ , & n^A_{\nu,f} = n^A_{\nu,i} \ \ , \\ n^N_{\pi,f} = n^N_{\pi,i} \ \ , & n^N_{\nu,f} = n^N_{\nu,i} - 2 \ \ . \end{array} \tag{2}$$

If they do not, the $\beta\beta_{2\nu}$ decay becomes forbidden. This is the first selection rule the pseudo SU(3) formalism impose to the double beta decay. As an example, for ^{150}Nd we have obtained the occupation numbers $n_\pi^A = 4,\ n_\pi^N = 6,\ n_\nu^A = 2,\ n_\nu^N = 6$.

We have selected the standard version of the pseudo SU(3) Hamiltonian[13]. It is constructed by a spherical Nilsson hamiltonian which describes the single-particle motion of neutrons or protons, a quadrupole-quadrupole interaction and a residual force. The latter allows the fine tuning of low lying spectral features like K band splitting and the effective moments of inertia.

With the occupation numbers and the hamiltonian discussed above, the wave function of the deformed ground state of ^{150}Nd can be written[8]

$$\begin{array}{rl} |^{150}Nd, 0^+\rangle = & |\ (h_{11/2})^4_\pi,\ J^A_\pi = M^A_\pi = 0;\ (i_{13/2})^2_\nu,\ J^A_\nu = M^A_\nu = 0 >_A \\ & |\ \{1^6\}_\pi\{2^3\}_\pi(12,0)_\pi;\ \{1^6\}_\nu\{2^3\}_\nu(18,0)_\nu;\ 1(30,0)K = 1 J = M = 0 >_N \ \ , \end{array} \tag{3}$$

3 The double beta decay

The inverse half life of the two neutrino mode of the $\beta\beta$-decay can be expressed in the form[14]

$$\left[\tau^{1/2}_{2\nu}(0^+ \rightarrow J^+_\sigma)\right]^{-1} = G_{2\nu}(J^+_\sigma)\ |\ M_{2\nu}(J^+_\sigma)\ |^2 \ \ . \tag{4}$$

where $G_{2\nu}(J^+_\sigma)$ are kinematical factors. They depend on $E_{J;\sigma} = \frac{1}{2}[Q_{\beta\beta} - E(J_\sigma)] + m_e c^2$ which is the half of total energy released. The nuclear matrix element is evaluated using the pseudo SU(3) formalism. For the $^{150}Nd \rightarrow {}^{150}Sm$ case it can be written as

$$
\begin{aligned}
M_{2\nu}^{GT}(J_\sigma^+) \; = \; & a(J)\; b(n_\pi^A)\; \mathcal{E}_{J\sigma}^{-(J+1)} \\
& \sum_{(\lambda_0\mu_0)K_0} < (0\tilde{\eta})1\tilde{l}, (0\tilde{\eta})1\tilde{l} \| (\lambda_0\mu_0)K_0 J >_1 \sum_{\rho} < (30,0)1\,0, (\lambda_0\mu_0)K_0 J \| (\lambda\mu)_\sigma 1 J >_\rho \\
& \sum_{\rho'} \left[\begin{array}{cccc} (12,0) & (0,0) & (12,0) & 1 \\ (18,0) & (\lambda_0\mu_0) & (12,2) & \rho' \\ (30,0) & (\lambda_0\mu_0) & (\lambda\mu)_\sigma & \rho \\ 1 & 1 & 1 & \end{array} \right] < (12,2) \; ||| \; [\tilde{a}_{0\tilde{\eta}),\frac{1}{2}} \tilde{a}_{0\tilde{\eta}),\frac{1}{2}}]^{(\lambda_0\mu_0)} \; ||| \; (18,0) >_{\rho'}
\end{aligned}
\tag{5}
$$

In the above formula $< \ldots, \ldots \|, , >$ denotes the SU(3) Clebsch-Gordan coefficients[15], the symbol [...] represents a $9 - \lambda\mu$ recoupling coefficient[16], $< .. \; ||| \; .. \; ||| \; .. >$ is the triple reduced matrix elements[17] and

$$
\begin{aligned}
& a(0) = \frac{4\eta}{(2\eta+1)\sqrt{2\eta-1}}, \quad a(2) = \frac{2}{2\eta+1}\left[\frac{5\eta(\eta-1))2\eta-3)}{3(2\eta+1)}\right]^{1/2}, \quad b(n_\pi^A) = [(n_\pi^A+2)(\eta+1-n_\pi^A/2)]^{1/2}, \\
& \mathcal{E}_{J\sigma} = E_{J\sigma} - \hbar\omega k_\pi 2 j_\pi + \Delta_C. \quad \Delta_C = \frac{0.70}{A^{1/3}}[2Z+1-0.76((Z+1)^{4/3}-Z^{4/3})]MeV \;\; .
\end{aligned}
\tag{6}
$$

The SU(3) tensorial componentes (λ_0, μ_0) of the normal part of the double Gamow-Teller operator must be able to couple the proton and total irreps (18,0) and (30,0) associated with the ground state of ^{150}Nd to the corresponding irreps (12,2) and $(\lambda\mu)_\sigma)$ which characterize the ground and excited rotational bands in ^{150}Sm. If it is not possible the $\beta\beta_{2\nu}$ decay to the 0^+ and 2^+ states of an excited band is forbidden. This is the second selection rule imposed by the model to the $\beta\beta_{2\nu}$ decay.

In the case the 0ν decay exists, the virtual neutrino must be emitted in one vertex, and absorbed in the other. Since in the standard theory the emitted particle is a right-handed antineutrino and the absorbed one a left-handed neutrino the process requires that a) the exchanged neutrino is a Majorana particle and b) both neutrinos have a common helicity component. The helicity matching can be satisfied in two ways: a) the neutrinos have a nonvanishing mass and therefore a “wrong” helicity component proportional to m_ν/E_ν. The decay rate will be proportional to $< m_\nu >^2$. Or b) the helicity restriction could be satisfied if there is a right handed current interaction. In this case a nonvanishing mass allowing mixing of neutrino types is also required [14, 18].

The $\beta\beta_{0\nu}$ matrix elements were evaluated using the pseudo SU(3) formalism[11]. The final expressions are similar to those related with the two neutrino mode, given in Eq. (5), but restricted to the decay to the ground state of the final nuclei. They explicitly include the "neutrino potential" originated by the interchange of a Majorana neutrino between the two emitting neutrons. The 0ν mode allows transitions with $\Delta l \neq 0$, whose contribution to the $\beta\beta_{0\nu}$ matrix elements were found an order of magnitude lesser than those which do not change the orbital angular momentum.

The five nuclei which were found to have the $\beta\beta_{2\nu}$ mode strictly forbidden within the pseudo SU(3) model are allowed to have $\beta\beta_{0\nu}$ decays because the neutrino potential allows parity mixing transitions. We would expect that these 0ν matrix elements, related with $\Delta l = 1, 3, 5$ transitions were of the same order of magnitude of the $\Delta l = 2, 4, 6$ transitions and about a factor ten lesser than the $\Delta l = 0$ matrix elements. This hindrance is comparable with the modifications that the symplectic model would introduce, allowing more than one shell active for each kind of nucleons. These questions are under current investigation.

4 Results and discussion

We predict, according to the assumptions of the model, that the two neutrino double beta transitions of the following five nuclei:

$$\begin{array}{ccc}
^{154}Sm & \rightarrow & ^{154}Gd \\
^{160}Gd & \rightarrow & ^{160}Dy \\
^{176}Yb & \rightarrow & ^{176}Hf \\
^{232}Th & \rightarrow & ^{232}U \\
^{244}Pu & \rightarrow & ^{244}Cm
\end{array}$$

are forbidden. In particular we predict a null result for the present search in ^{244}Pu[6], a potential strong test of our model. Inclusion of pairing within the same truncation scheme will allow mixing of different pseudo SU(3) irreps, and is in progress[19]. The lack of a spin-isospin channel in our Hamiltonian is perhaps an additional limitation, but it is included, at least partially, in the quadrupole-quadrupole interaction, when it is recoupled.

In Table 1 are indicated the results for the six $\beta\beta$ emitters in which we get a $\beta\beta_{2\nu}$ matrix element different from zero[8]. The value for the integrated kinematical factors were obtained following the procedure indicated by Doi et al[20] with $g_A/g_V = 1.0$. In the second and third columns the theoretical and experimental $\beta\beta_{2\nu}$-half lives are given. The agreement with the available data for ^{150}Nd and ^{232}U is good. Improvements in the experimental information about the decay of these nuclei together with ^{148}Nd is possible, but the other three are almost excluded of detection because their lepton kinematical factors $G_{2\nu}$ are very small. This is particularly true for ^{146}Nd, which has the smallest Q value for the $\beta\beta$ decay and thus becomes strongly inhibited.

We also evaluated the $\beta\beta_{0\nu}$ matrix elements between the the same six nuclei[10]. In the last two columns of Table 1 the theoretical predictions and experimental lower limits of the $\beta\beta_{0\nu}$-half lives are given. In order to calculate the zero-neutrino half-life we assumed $< m_\nu >= 1eV$, which is the larger neutrino mass parameter compatible with the ^{76}Ge experiment [21]. In the case of ^{238}U the predicted 0ν half life is three orders of magnitude greater than the predicted 2ν half life, which essentially agrees with the experimental one, confirming that the observed $\beta\beta$ decay of ^{238}U has to be the two neutrino mode. The transition is strongly dominated by a pair of neutrons in the normal parity orbital $i^{\nu}_{11/2}$ decaying into two protons in the unique orbital $i^{\pi}_{13/2}$,

In the case of ^{150}Nd, the pseudo SU(3) 0ν matrix element reported here is about a factor four lesser than the QRPA estimations[9]. This is a very relevant result. First, it exhibits the stability of the neutrinoless double beta decay matrix elements evaluated in quite different nuclear models, in the case of deformed nuclei. Second, this factor of four, which is little compared with the order of magnitude variations in the 2ν theoretical estimations, is still important in order to extract the parameter $< m_e >$. In this case the transition is dominated by a pair of neutrons in the normal parity orbital $h^{\nu}_{9/2}$ decaying into two protons in the unique orbital $h^{\pi}_{11/2}$, again resembling the two neutrino case.

As can be seen in the last two columns of Table 1, the $\tau^{1/2}_{0\nu}$ predicted for $< m_\nu >= 1eV$ are at least three order of magnitude greater than the experimental limits. These results reflect the fact that at, the present stage of the experimental $\beta\beta$ research, the limits $< m_\nu >\leq 1.1eV$ obtained by the Heidelberg-Moscow collaboration [21] using significative volumes of ultrapure

^{76}Ge are the most sensitive. But, if the $\beta\beta_{0\nu}$ decay is observed in ^{76}Ge, at least a second observation will be essential, and ^{150}Nd is a likely candidate to do this job [4, 18]. In the next few years the limit for $< m_\nu >$ extracted from $\beta\beta_{0\nu}$ experiments is expected to be improved up to 0.1 eV and ^{150}Nd is one of the selected isotopes [22].

Finally we review the two neutrino mode of the double beta decay $\beta\beta_{2\nu}$ of ^{150}Nd into the ground state, the first excited 2^+ and the first and second excited 0^+ states of $^{150}Sm^{150}Sm$[11].

In Table 2 the matrix elements and predicted half-lives for the $\beta\beta_{2\nu}$ decay of ^{150}Nd to the ground state, the first 2^+ and the first and second excited 0^+ states of ^{150}Sm are presented. The matrix elements are given in units of $(m_e c^2)^{-(J+1)}$. It must be mentioned that the phase space factors differ in about 10% with those presented in Table 1[8] where a different renormalization procedure was used.

The $\beta\beta_{2\nu}$ decay to the first excited 0^+ state is *forbidden.* In this model this is imposed by the fulfilment of an exact selection rule. The pair of annihilation operators $\tilde{a}_{(0\tilde{4})\frac{1}{2}}$, when expanded in their SU(3) components, cannot couple the ^{150}Nd g.s. irrep (30,0) to the irrep (20,4) which we associated with the first excited 0^+ state. Thus the transition between members of these particular irreps are forbidden.

The decay to the second excited 0^+ state is allowed but strongly cancelled. The predicted half-life is four orders of magnitude larger than that of the decay to the g.s. The $\beta\beta_{2\nu}$ decay to the 2^+ state is inhibited by the μ_N^3 dependence of the matrix element as it is discussed in Section 3. The matrix element of the $\beta\beta_{2\nu}$decay to the first excited 2^+ state $M_{2\nu}(2^+_{g.s.})$ is three orders of magnitude lesser than the matrix element of the decay to the g.s. $M_{2\nu}(0^+_{g.s.})$.

The present results contradicts those previously published[23, 24] were it was speculated that the $\beta\beta_{2\nu}$ decay of ^{150}Nd to the first excited 0^+ state of ^{150}Sm could have a similar intensity of that to the g.s. We found that in the present formalism this decay is forbidden. If we select different occupation numbers for both ^{150}Nd and ^{150}Sm, taken the deformation of the latter nucleus instead of that of the former, we found very similar results for the decay to the g.s. and the first 2^+ state, but the matrix elements of the decay to the first and second 0^+ states becomes interchanged with essentially the same values. Considering the difference in the phase space integrals we predict a half live of the order 10^{21} years for the decay to the first excited 0^+ state and the decay to the second excited one becomes forbidden.

The above discussed reduction of the matrix element of the $\beta\beta_{2\nu}$ decay to the excited 0^+ state as compared with the decay to the g.s. is not a general result of the pseudo SU(3) scheme. A recent analysis of the case of ^{100}Mo[25] shows that both matrix elements are very similar and that they are in agreement with the experimental information. In conclusion, the appearance of selection rules which can produce the suppression of the matrix elements governing a $\beta\beta_{2\nu}$ transition is a consequence of the details of the irreps involved.

5 Conclusions

The pseudo SU(3) model is a very powerful machinery to describe the collective behavior of heavy deformed nuclei. It has been used to reproduce very accurately the rotational spectra of heavy deformed nuclei, including the K-band splitting, the amplitudes for transitions of the E2, M1 and M3 type.

We have used this model to evaluate the $\beta\beta_{2\nu}$ half lives of 11 heavy deformed nuclei, using the pseudo SU(3) approach together with a summation method, which avoids the closure approximation, obtaining good agreement with the available data and making testable predictions, quite different from the QRPA ones in some cases.

We have also evaluated the $\beta\beta_{0\nu}$ half lives of six of these heavy deformed nuclei. We obtained an order of magnitude agreement with the QRPA estimations, and a factor four of difference. We exhibited predictions for the neutrinoless double beta half-lives assuming $< m_\nu >= 1eV$, and discussed the relevance of our results. In the case of ^{238}U this result complements those obtained for the two neutrino $\beta\beta$ decay, and it confirms the observed half-life as two neutrino in origin.

At last we have studied the $\beta\beta_{2\nu}$ decay mode of ^{150}Nd to the ground and excited states of ^{150}Sm. The $\beta\beta_{2\nu}$ decay to the first excited 0^+ state was found forbidden in the model and the decay to the second excited 0^+ state has a half-life four orders of magnitude greater than that to the g.s.. The decay to the 2^+ state is strongly inhibited due to the energy dependence of the matrix elements $M_{2\nu}(2^+)$, two powers greater than that of the matrix element $M_{2\nu}(0^+)$.

In all the calculations only one active shell was allowed for protons, and one for neutrons.

This is a very strong truncation, of the same type as used in shell model calculations. An important consequence of this truncation is the fact that only one uncorrelated Gamow-Teller transition is allowed: that which removes a neutron from a normal parity state with maximum angular momentum, and creates a proton in the intruder shell ($h^{\nu}_{9/2} \rightarrow h^{\pi}_{11/2}$ in rare earth nuclei, $i^{\nu}_{11/2} \rightarrow i^{\pi}_{13/2}$ in actinides). This unique Gamow-Teller transition controls the double beta decay. If the occupation of the Nilsson levels is such that the number of protons in the abnormal states does not change going from the initial to the final state configurations, the decay becomes forbidden.

The pseudo SU(3) model uses a quite restrictive Hilbert space. The model could be improved by incorporating mixing between different irreps, via pairing by example[19]. Also other active shells can be taken into account in the symplectic extension[27]. In both cases the selection rules that impose such strong restrictions to the $\beta\beta_{2\nu}$ decays of some nuclei can be superseded. However if the main part of the wave function is well represented by the pseudo SU(3) model those forbidden decays will have, in the better case, matrix elements that will be no greater than 20% of the allowed ones, resulting in at least one order of magnitude cancellation in the half-life.

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Table Captions

Table 1. Theoretical estimates for the $\beta\beta$-decay half-life in the 2ν and 0ν mode for several heavy deformed nuclei are given and compared with the available experimental data.

Table 2 The dimensionless matrix elements and predicted half-lives for the $\beta\beta_{2\nu}$ decay of ^{150}Nd to the ground state, the first 2^+ and the first and second excited 0^+ states of ^{150}Sm.

Table 1

Transition	$\tau_{2\nu}^{1/2}$ $[yr]$ *theo*	*exp*	$\tau_{0\nu}^{1/2}$ $[yr]$ *theo*	*exp*
$^{146}Nd \rightarrow^{146} Sm$	2.1×10^{31}		1.18×10^{28}	
$^{148}Nd \rightarrow^{148} Sm$	6.0×10^{20}		6.75×10^{24}	
$^{150}Nd \rightarrow^{150} Sm$	6.0×10^{18}	$9 - 17 \times 10^{18}$ [2, 4, 5]	1.05×10^{24}	$> 2.1 \times 10^{21}$ [4]
$^{186}W \rightarrow^{186} Os$	6.1×10^{24}		5.13×10^{25}	$> 2.3 \times 10^{20}$ [26]
$^{192}Os \rightarrow^{192} Pt$	9.0×10^{25}		3.28×10^{26}	
$^{238}U \rightarrow^{238} Pu$	1.4×10^{21}	2×10^{21} [3]	1.03×10^{24}	$> 2.0 \times 10^{21}$ [3]

Table 2

	$M_{2\nu}^{GT}(J_\sigma^+)$	$\tau_{2\nu}^{1/2}(0^+ \rightarrow J_\sigma^+)[yr]$
$0^+ \rightarrow 0^+(g.s.)$	.0549	6.73×10^{18}
$0^+ \rightarrow 0^+(1)$	0	∞
$0^+ \rightarrow 0^+(2)$	.00499	4.31×10^{22}
$0^+ \rightarrow 2^+$	5.38×10^{-5}	7.21×10^{24}